Stage 9 PROMPT Sheet

9/1 Standard Form

Standard form is a way of expressing numbers that are too big or too small to be conveniently written in decimal form

$$A \times 10^n$$

'A' is between 1 & 10 and 'n' is an integer

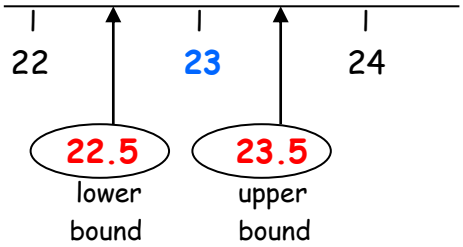
Example: 93 000 000 can be written 9.3×10^7

0.00000345 can be written 3.45×10^{-6}

9/2 Express error intervals when rounding

Error interval – the range of values (between the upper and lower bounds) in which the precise value could be

- If 23cm is rounded to nearest whole cm
- 23 is between the whole numbers 22 & 24

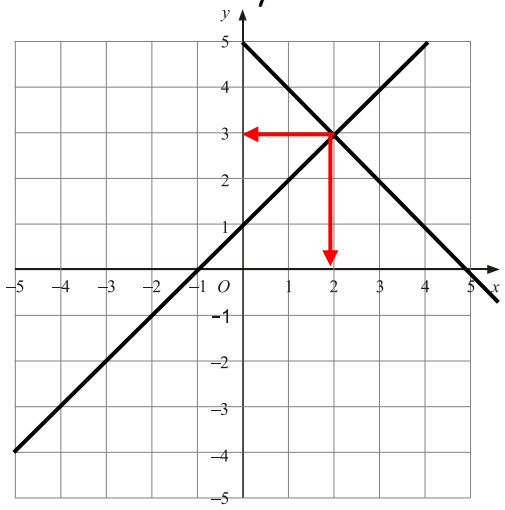


Inequality notation to specify error interval:

$22.5 < x \leq 23.5$

9/3 Solve simultaneous equations graphically

- Draw the graphs of the equations
- Find where they cross



Solution is x = 2, y = 3

9/3 Solve simultaneous equations by an algebraic method

- Make the number of ys the same
- Add or subtract to eliminate the ys
  Same signs ~ subtract
  Different signs ~ add
- Find the value of x
- Substitute the value of x to find y

Example: $2x - 3y = 11$ (x2)

$5x + 2y = 18$ (x3)

$4x - 6y = 22$

$15x + 6y = 54$

Add the two equations to eliminate y

$19x = 76$

$x = 4$

Substitute x = 4 into one of the equations

$5x + 2y = 18$

$5x4 + 2y = 18$

$20 + 2y = 18$

$2y = -2$

$y = -1$

9/4 Factorise quadratic expressions

* Form $x^2 + bx + c$

$x^2 - 3x - 4 = (x - 4)(x + 1)$

$x^2 - 25 = (x - 5)(x + 5)$

Difference of 2 squares

Solve quadratic equations by factorising

Example: To solve $x^2 = 6x - 5$

- **Put equation in form $x^2 + bx + c = 0$**

$x^2 - 6x + 5 = 0$

- **Factorise the left hand side**

$(x - 5)(x - 1) = 0$

- **Equate each factor to zero**

$x - 5 = 0$ or $x - 1 = 0$

$x = 5$ or $x = 1$

9/5 Solve inequalities

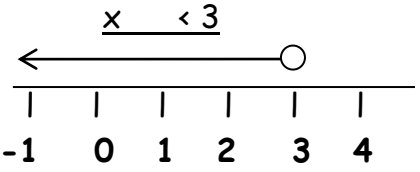
Inequalities can be treated like equations

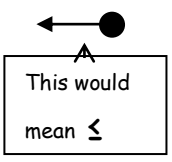
The solution can be shown on a number line

e.g.1 $2x - 4 < 2$ *(+4 to each side)*

$2x < 6$ *(÷2 each side)*

$x < 3$





9/6 Rearrange a formula

Isolate the required new subject by balancing

Example: To make 'y' the subject of $3x + 2y = 7$

$3x + 2y = 7$ (-3x both sides)

$2y = 7 - 3x$ (÷2 both sides)

$y = \frac{7 - 3x}{2}$

or $y = 3.5 - \frac{3}{2}x$

To get rid of:

+ ⟷ −

× ⟷ ÷

$\sqrt{\ }$ ⟷ $(\ )^2$

9/7 Solve equations with fractions

Transform it into an equation **without** fractions

Use the balancing technique

Example: To solve $\frac{x}{3} + \frac{x-2}{5} = 6$

Multiply both sides by 15 (LCM of 3 & 5)

⇨ $5x + 3(x - 2) = 90$

⇨ $5x + 3x - 6 = 90$

⇨ $8x - 6 = 90$

⇨ $8x = 96$

⇨ $x = 12$

9/8 Use y=mx+c to identify parallel lines

Parallel lines have the same gradient

Example: $y = 3x - 1$

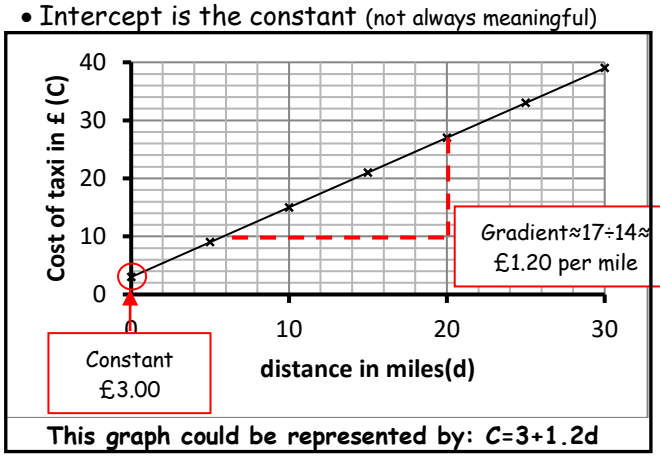
$y = 3x$

$y = 5 + 3x$

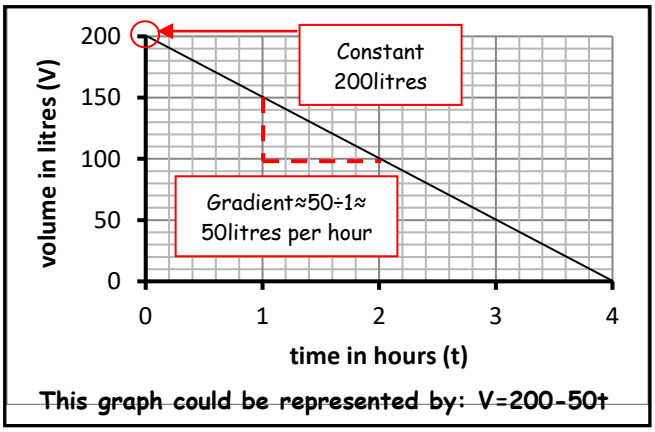
'm' represents the gradient
All parallel because they have the same gradient of 3

9/9 Interpretation of gradient & intercept

- Gradient (slope) is the rate of change
- Intercept is the constant (not always meaningful)



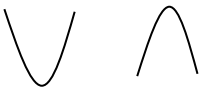
This graph could be represented by: C=3+1.2d



This graph could be represented by: V=200-50t

9/10 Plot & use quadratic graphs

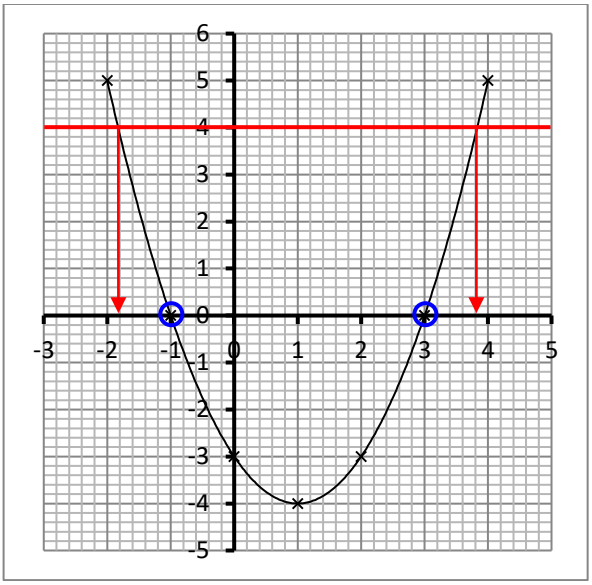
The shape of a quadratic graph is a parabola



You will need to complete a table of values to work out the points to plot

Example: $y = x^2 - 2x - 3$

x	-2	-1	0	1	2	3
y	5	0	-3	-4	-3	0



- **To solve $x^2 - 2x - 3 = 0$**

Note down the x-values where the graph cuts the x-axis (y=0) **i.e. x = -1 and 3**

- **To solve $x^2 - 2x - 3 = 4$**

Note down the x-values where the graph cuts the line y = 4 **i.e. x ≈ -1.8 and 3.8**

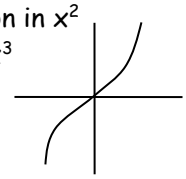
9/11 Recognise & sketch graphs

- Learn the basic shapes of graphs

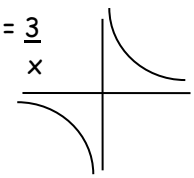
Linear graphs – **straight line** - equation in x

Quadratic graph – **parabola** – equation in x^2

Cubic graph – **S-shape** – equation in x^3



Reciprocal graph – **see sketch** e.g $y = \frac{3}{x}$



9/12 Substitute into a formula/expression

Remember ~ the rules for integers
~ BIDMAS for order of operations

Examples:

3 + (-4) = 3 - 4 = -1
3 - (-4) = 3 + 4 = 7

(-6) x (-2) = +12
(6) x (-2) = -12

+ - = -
- - = +
+ x - = -
- x - = +

Example: Evaluate $P = x^2 - 7x$ when $x = -5$

$P = (-5)^2 - 7x$
$P = (-5) \times (-5) - 7 \times (-5)$
$P = +25 + 35$
$P = 60$

SUVAT formulae will be given:

$v = u+at$
$s = ut + \frac{1}{2} at^2$
$v^2 = u^2 + 2as$

s=displacement; u=initial velocity
v=final velocity; a=acceleration; t=time

Whenever you have a SUVAT question, identify the **three** things you know and the **one** thing you want to find out. Use the equation with these four things in.

- *If something is starting from rest then the initial velocity (u) is zero*
- *acceleration - gravity applies to all falling objects approximately 10m/s²*

9/13 Simple interest

Example:
£350 in a bank for 3 years at a rate of 8% per annum simple interest. Find the interest/balance.
After 1 year, interest = 0.08 x 350
After 3 years, interest = 0.08 x 350 x 3 = £84
Balance = £350 + £84 = £434

Compound interest

Example:
£350 in a bank for 3 years at a rate of 8% per annum compound interest. Find balance.
Balance = 350×1.08^3 = £440.90

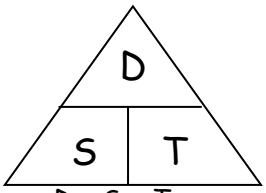
9/14 Find original value after % change

Remember:
To increase an amount by 15% you x 1.15
To decrease an amount by 15% you x 0.85

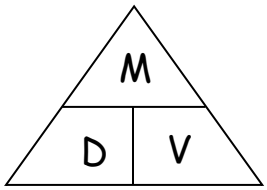
Example
New value = Original x %increase/decrease
e.g. New value = £12000 x 1.15 = £13800
Original value = New value ÷ %increase/decrease
Original value = 13800 ÷ 1.15 = £12000

9/15 Compound Measure – Learn formulae

- These triangles are useful
- Cover the quantity you are trying to find
- What is uncovered is the formula to use



D = S x T
S = D ÷ T
T = D ÷ S

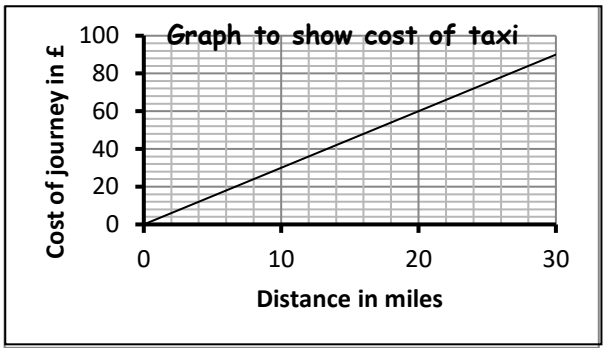


M = D x V
D = M ÷ V
V = M ÷ D

9/16 Direct and inverse proportion

- **Direct proportion**

~As one amount **increases**, another amount **increases** at the same rate.
~The graph is a straight line with a positive slope AND goes through the point (0, 0)
~The two variable divided give a constant
e.g. **As distance increases, cost increases**

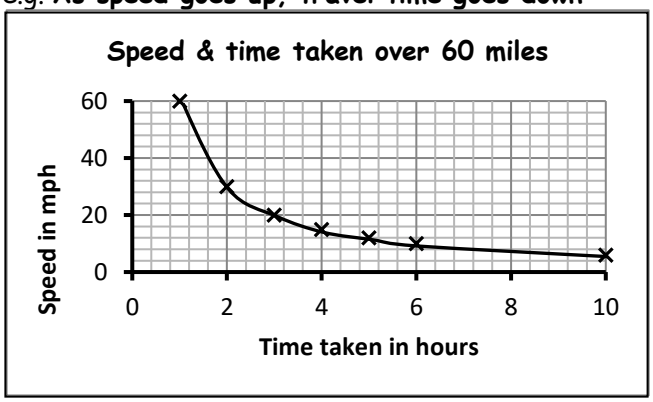


Gradient = cost ÷ distance = constant = 3
i.e £3 per mile

- **Inverse Proportion**

~As one amount **increases**, another amount **decreases** at the same rate.
~Inverse proportion leads to curved graphs
~The two variable multiplied give a constant

e.g. **As speed goes up, travel time goes down**



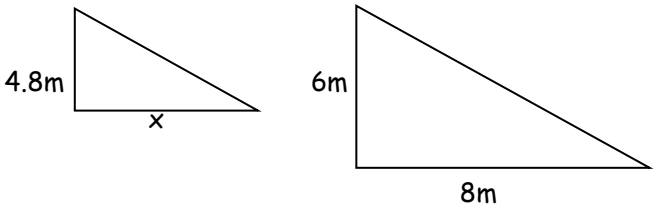
Time x Speed = constant = 60
i.e. 60miles

9/17 Missing lengths in similar shapes

If one shape is an enlargement of the other, we say they are similar.

- Corresponding angles are equal
- Corresponding sides have proportional lengths

Example – these 2 triangles are similar



Scale factor = 6 ÷ 4.8 = 1.25
x = 8 ÷ 1.25 = 6.4cm

N.B.
Always draw the 2 triangles separately and the same way up – it is easier to spot the sides that correspond to each other

9/18 Arc lengths & perimeters

Semi-circle
Perimeter = $\frac{1}{2}\pi d + d$

quarter circle
Perimeter = $\frac{1}{4}\pi d + 2r$

Sometimes answers can be in terms of π
e.g. 3π; 3π + 6

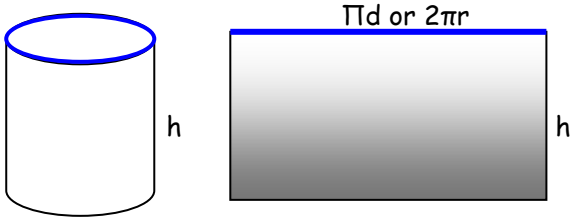
9/19 Sector areas

Semi-circle
Area = $\frac{1}{2}\pi r^2$

quarter circle
Area = $\frac{1}{4}\pi r^2$

9/20 Surface area of cylinder – Learn

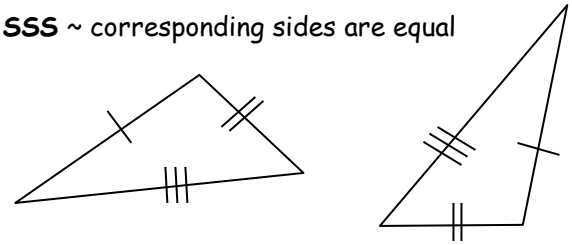
Curved Surface Area of cylinder = 2πrh



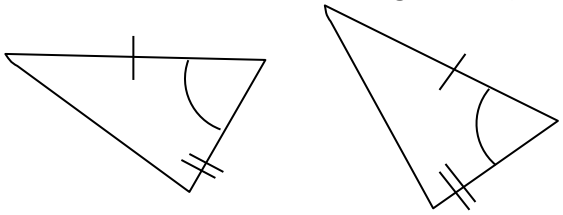
9/21 Congruence criteria

- **Congruent shapes have the same size and shape, one will fit exactly over the other.**
- **2 triangles are congruent if any of these 4 conditions are satisfied on each triangle**

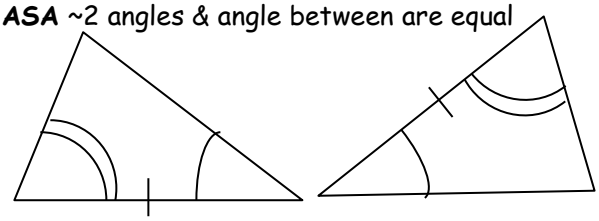
~ **SSS** ~ corresponding sides are equal



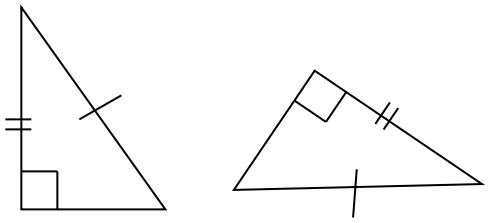
~ **SAS** ~2 sides & the included angle are equal



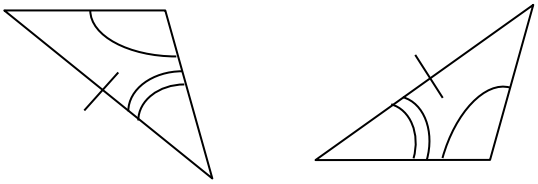
~ **ASA** ~2 angles & angle between are equal



~ **RHS** ~ Both triangles are right-angled, hypotenuses are equal and another pair of sides are equal

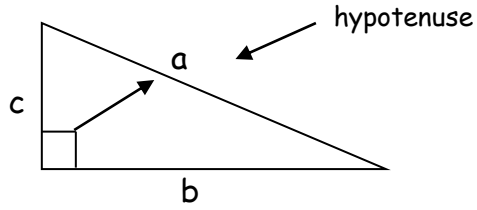


~AAS ~ 2 adjacent angles are equal & an angle excluding the included angle (similar to ASA because 3rd angle could be found)



9/22 Pythagoras Theorem - Learn

For this right angled triangle:



$a^2 = b^2 + c^2$

To find the hypotenuse

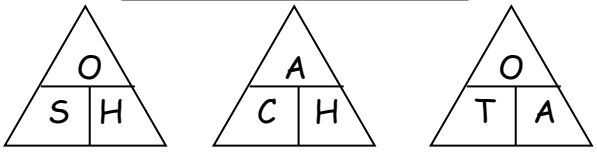
- ADD the squares of the other 2 sides
- Then square root the answer

To find one of the shorter sides

- SUBTRACT the squares of the other 2 sides
- Then square root the answer

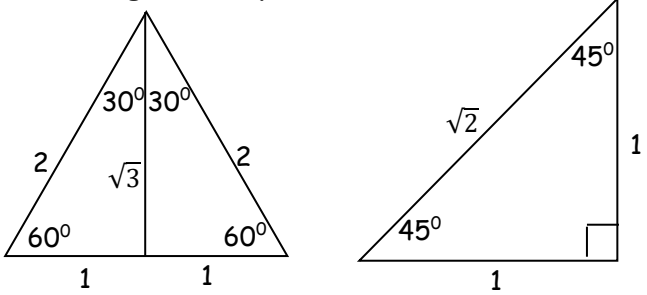
9/23 Trigonometry - Learn

SOH CAH TOA



EXAMPLES		
$\sin x = \frac{4}{5}$ $\sin x = 0.8$ $x = \sin^{-1}(0.8)$ $x = 53.1^\circ$	$\cos 28^\circ = \frac{x}{5}$ $x = 5 \times \cos 28^\circ$ $x = 4.4$	$\tan 28 = \frac{5}{x}$ $x = \frac{5}{\tan 28}$ $x = 9.4$

9/24 Trigonometry-know exact values

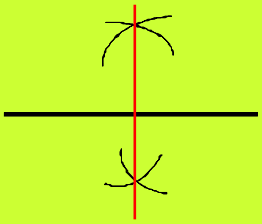


Angle	sin θ⁰	cos θ⁰	tan θ⁰
0	0	1	0
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
45	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	1	0	

9/25 Constructions of bisectors

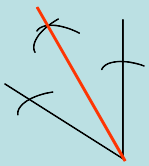
- Perpendicular bisector of a line

Draw a straight line through where the arcs cross above and below.



- Bisector of an angle

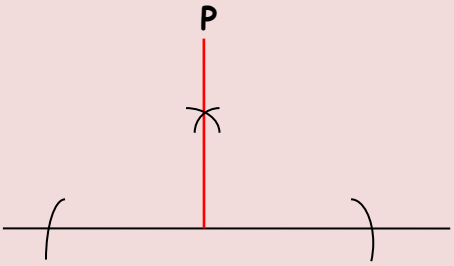
Draw a line from where the arcs cross to the vertex of the angle



9/26 Construction of perpendiculars

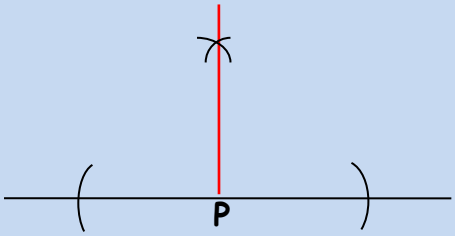
- Perpendicular **from** a point to a line

Draw arcs on the line from the point P



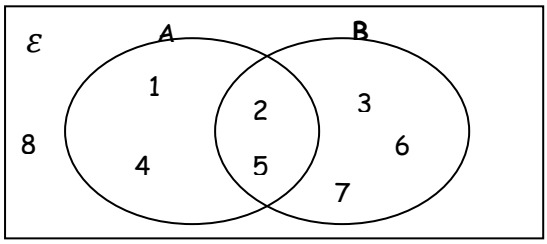
- Perpendicular **to** a point on a line

Draw arcs on the line from the point P



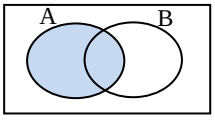
9/27 Venn Diagrams & Probability

Universal set ε= {1,2,3,4,5,6,7,8}

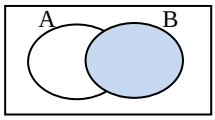


A = {1, 2, 4, 5}

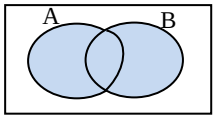
$2 \epsilon A$ (2 is a member of Set A)



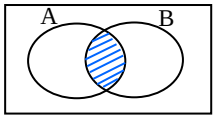
B = {2, 3, 5, 6, 7}



A∪B = {1, 2, 3, 4, 5, 5, 6, 7} (union)



A∩B = {2, 5) (intersection)



9/28 Tree Diagrams

- When going along the branches.

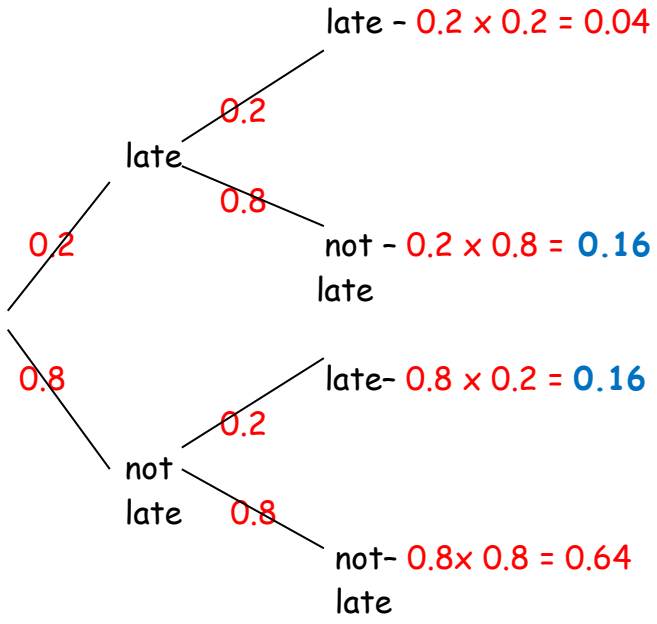
MULTIPLY the probabilities

- If more than one route is wanted,

ADD the probabilities

Example: The probability that Jane is late = 0.2

Day 1 **Day 2**



To find the probability of late on only one day:

day1	&	day2	OR	day1	&	day 2
late		not late		not late		late

= 0.16 + 0.16

= 0.32

9/29 Sampling

A sample is a subset containing the characteristics of a larger population & should be unbiased.

The larger the sample size the lower the risk of a non-representative sample

Sampling is a process of selection from a population of interest so that by studying the sample, the results can be extended to the population of interest.

- **Random sample** - each member has an equal chance of being chosen.

e.g. 30 chosen from a hat from a year group.

- **Systematic sample** – each member is selected according to a random starting point and a fixed, periodic interval.

e.g. Year group numbered 1-150. Start at 3 and choose every 10th from there.

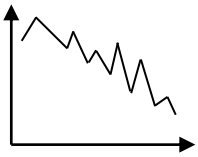
- **Stratified sample** – ensure sub-groups are equally represented and not biased.

e.g. for a sample of **30** from 90 boys and 60 girls in a year group, choose $\frac{90}{150} \times \mathbf{30} = 18\text{boys}$ randomly.

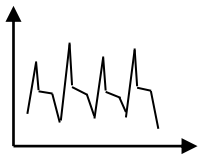
9/30 Time Series

Data collected over a period of **time** is called **time series data** and is used to form a **time series graph**. Time series graphs make trends easy to spot and can be used to project into the future.

- **Trend** is when, on average, measurements tend to increase (upward trend) or decrease (downward trend) over a period of time.



- **Seasonality** is indicated by a regular repeating pattern of highs and lows e.g. monthly, quarterly etc



- **Cyclic** is indicated by highs and lows but not over fixed periods of time